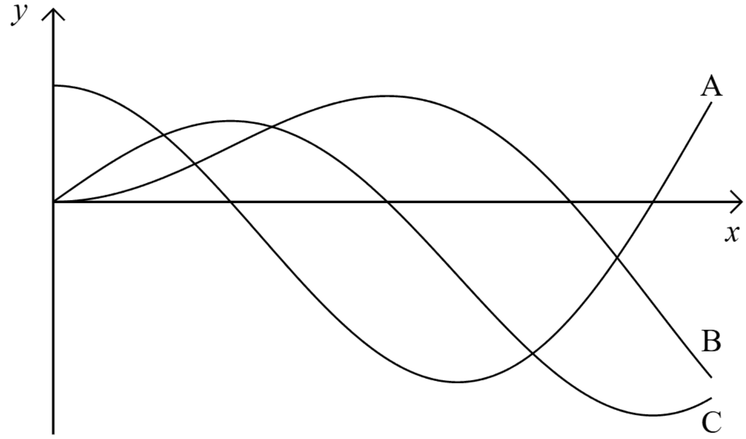
Name: ....

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E
23	A	B	C	D	E
24	A	B	C	D	E
25	A	B	C	D	E
26	A	B	C	D	E
27	A	B	C	D	E
28	A	B	C	D	E
29	A	B	C	D	E
30	A	B	C	D	E

1. The diagram below shows the graphs of $y = f(x)$, $y = f'(x)$ and $y = f''(x)$.



In the order written above, the graphs are

(A) A, B, C (B) B, A, C (C) C, A, B (D) B, C, A (E) A, C, B

---

2. The value of $\lim_{x \to 0} \frac{1}{2x} \times \lim_{x \to 0} 3x$ is

(A) 0 (B) $\frac{3}{2}$ (C) 1

(D) DNE (E) none of these

---

3. The minimum value of the function $f(x) = \frac{x}{\ln x}$ on the interval [2,4] is

(A) $\ln 2$ (B) e (C) $\ln 4$

(D) e^{-1} (E) none of these

---

4. The normal line to the parabola $y = x^2 - 4x + 1$ at $x = 3$ intersects with the parabola again when the value of x is equal to

(A) 1 (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) -2 (E) $\frac{1}{2}$

---

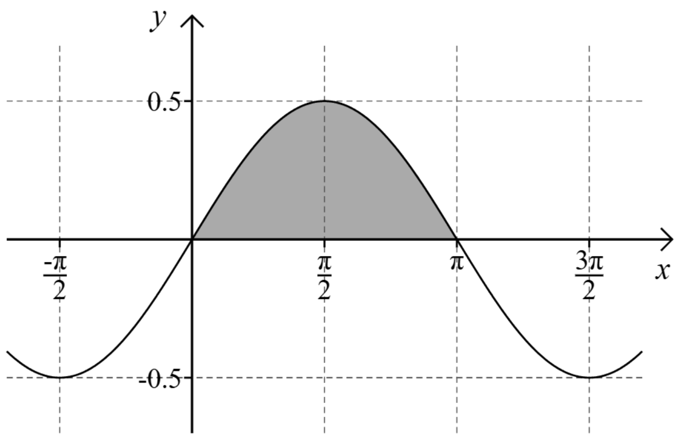
5. The average value of the function $f(x) = \sqrt{1 - x^2}$ is

(A) $\frac{1}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$

(D) $\frac{2}{3}$ (E) none of these

---

6. The diagram below shows the region completely bound by the graph of $y = f(x)$ and the x-axis on the interval $[0,\pi]$.



The area of the shaded region is equal to

(A) $\frac{1}{2}\int_0^{\pi} \cos x \, dx$ (B) $\int_0^{\sqrt{\pi}} x \sin(x^2) \, dx$ (C) $\frac{1}{2}\int_0^{\sqrt{\pi}} \sin(x^2) \, dx$

(D) $\int_0^{\pi} \sin x \, dx$ (E) none of these

---

7. If the area of a circle increases at a rate of 10 cm/s and the instantaneous rate of change of the circumference is equal to 1 cm/s then the radius is equal to

(A) π cm (B) 0.1 cm (C) 1 cm (D) 10 cm (E) 5 cm

---

8. The value of $\lim_{n \to \infty} \sum_{r=0}^{\infty} \frac{e-1}{n} \cdot \ln\left(1 + \frac{r(e-1)}{n}\right)$ is

(A) ∞ (B) e (C) 1 (D) -1 (E) 0

---

9. The value of $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ is

(A) 0 (B) ∞ (C) 3 (D) 6 (E) DNE

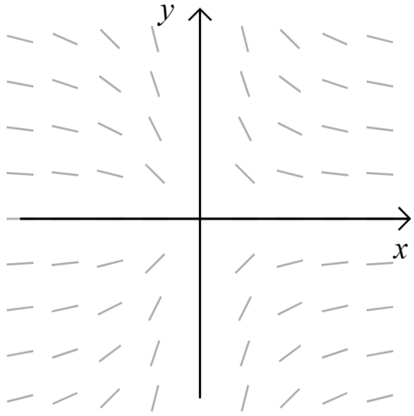
---

10. The maximum value of the product of two numbers with a sum of 300 is

(A) 450 (B) 225 (C) 200 (D) 375 (E) 275

---

11. The diagram below shows the slope field for a differential equation.



Which of the following represents a possible particular solution to the equation?

(A) $y = e^{-x}$ (B) $y = e^{1/x}$ (C) $y = e^{x^2}$ (D) $y = e^{-1/x}$ (E) $y = e^{-x^2}$

---

12. The curve $x \sin y = y \cos x$ has vertical tangents when

(A) $\cos x = x \cos y$ (B) $\sin y = y \cos x$ (C) $\cos x = y \cos y$

(D) $\sin x = x \cos y$ (E) $\cos x = y \sin y$

---

13. The curve $x \ln y = x + y$ at $x = 10$ has a vertical tangent at

(A) $(0,0)$ (B) (e,e) (C) $(1,1)$ (D) (e^2,e^2) (E) (e^{-1},e^{-1})

---

14. The function $f(x) = xe^{1-\sqrt{x}}$ is increasing on which interval?

(A) $0 < x < 8$ (B) $x > 1$ (C) $4 < x < 8$ (D) $0 < x < 4$ (E) $0 < x < 1$

---

15. Using the substitution $u = \sec x + \tan x$ the definite integral $\int_0^{\pi/3} \sec x \, dx$ is equal to

(A) 1 (B) $\ln\left(\frac{\pi}{3}\right)$ (C) $\ln\left(2+\sqrt{3}\right)$

(D) $\ln 2$ (E) $\frac{\ln 3}{2}$

---

16. The velocity of a particle in m/s after t seconds is given by $v(t) = 2t^{1/3} - t^{1/2}$. The speed of the particle in m/s after t seconds is given by

$$v_s(t) = \begin{cases} 2t^{1/3} - t^{1/2} & \text{for } 0 \le t < k \\ t^{1/2} - 2t^{1/3} & \text{for } t \ge k \end{cases}$$

The value of k is

(A) 16 (B) 32 (C) 64 (D) 128 (E) 256

---

17. If $e^{xy} = x + y$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{1+ye^{xy}}{1+xe^{xy}}$ (B) $\frac{1-e^{xy}}{xe^{xy}-1}$ (C) $\frac{1-ye^{xy}}{xe^{xy}-1}$ (D) $\frac{1-ye^{xy}}{1+xe^{xy}}$ (E) $\frac{1-e^{xy}}{1+xe^{xy}}$

---

18. If $f(x) \le g(x) \le h(x)$ for all x in the domain of $g(x)$, where $f(x)$ and $h(x)$ are continuous over $\mathbb{R}$, then $g(x)$ must be discontinuous over $\mathbb{R}$ when

(A) $f(x) = 2x - 1$ and $h(x) = x^2$

(B) $f(x) = -2 - 2x$ and $h(x) = x^2 - 1$

(C) $f(x) = -x^2$ and $h(x) = x^2$

(D) $f(x) = 4x - 5$ and $h(x) = 2x^2 - 3$

(E) none of these guarantee $g(x)$ is discontinuous over $\mathbb{R}$

---

19. Consider the differential equation

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

for some function f of $\frac{y}{x}$.

Which of the following substitutions will convert this into a separable differential equation?

(A) $x = uy$ (B) $u = xy$ (C) $y = ux$ (D) $u = x^2 y$ (E) $u = xy^2$

---

20. If $f(x) = \sec^{-1} x$ then $f'(x)$ is equal to

(A) $\dfrac{1}{\sqrt{x^2(x^2-1)}}$ (B) $\dfrac{1}{\sqrt{x^2-1}}$ (C) $-\dfrac{1}{\sqrt{x(x^2-1)}}$

(D) $\dfrac{1}{\sqrt{x^2(1-x^2)}}$ (E) $\dfrac{1}{\sqrt{x(1-x^2)}}$

---

21. The table below shows the values of a continuous function $f(x)$ for various values of x.

x	1	3	6
$f(x)$	2	k	7

For which of the following values of k does the equation $f(x) = 8$ have at least two solutions in the interval $(1,6)$?

I. 8

II. 6

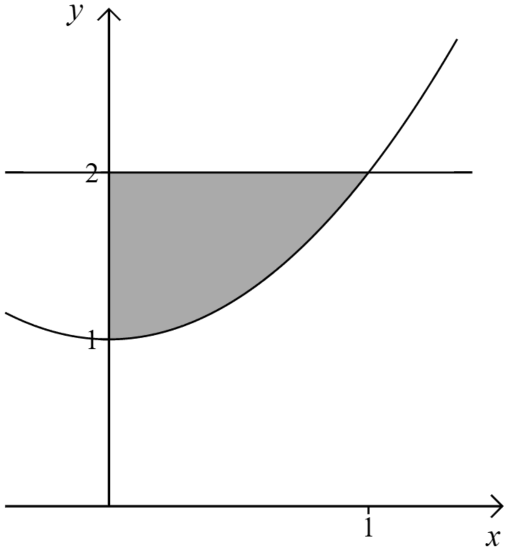
III. 9

(A) II and III only (B) III only (C) I and II only

(D) I only (E) I and III only

---

22. The diagram below shows the region completely bound by the graph of $y = x^2 + 1$, the line $y = 2$, and the y-axis.



The region is rotated 360° about the y-axis. The volume of the solid formed is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{5}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$

---

23. The value of $\int_0^1 \frac{1}{x^2 + 4x + 5}\,dx$ is

(A) $\arctan 2 - \frac{\pi}{4}$ (B) $\arctan 3 - \arctan 2$ (C) $\frac{\pi}{4}$

(D) $\arctan 3 - \frac{\pi}{4}$ (E) none of these

---

24. The table below shows the values of $\int_1^k f(x)\,dx$ for various values of k.

k	2	3	6
$\int_1^k f(x)\,dx$	4	8	16

The value of $\int_2^6 f\left(\frac{x}{2}\right)dx$ is equal to

(A) 8 (B) 4 (C) 16

(D) 12 (E) none of these

---

25. A particular solution to the differential equation $\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$ is

(A) $y = x(10 + 4\ln x)^{1/3}$ (B) $y = x(10 + 3\ln x)^{1/3}$ (C) $y = x(10 + 2\ln x)^{1/3}$

(D) $y = x(10 + \ln x)^{1/3}$ (E) $y = x(10 - \ln x)^{1/3}$

---

26. The value of $\int_{-4}^{0} \frac{x^2}{x+5}\,dx$ is

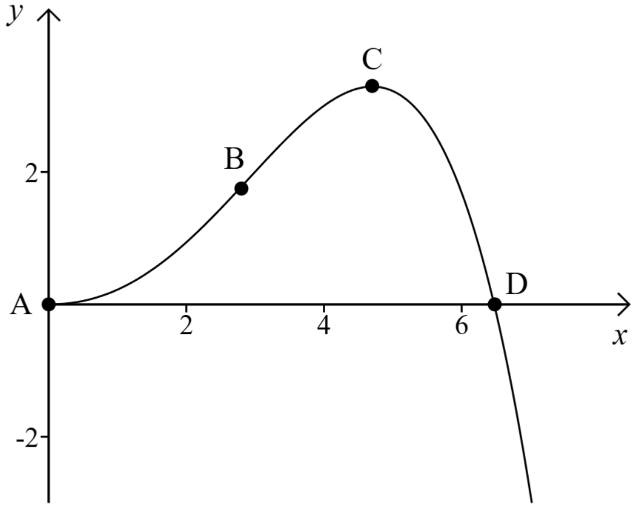
(A) $25\ln 5 - 28$ (B) $20\ln 5 - 28$ (C) $15\ln 5 - 28$

(D) $10\ln 5 - 28$ (E) $5\ln 5 - 28$

---

27. Let $g(x)=\int_0^x f(t)\,dt$ for a continuous function $f(x)$.

The diagram below shows the graph of $y=g(x)$. There is a point of inflection at B.



Which point corresponds to an x-intercept on the graph of $y=f(x)$?

(A) A (B) B (C) C

(D) D (E) None of these

---

28. The limit $\lim_{h\to 0}\dfrac{\ln(x-h)-\ln x}{h}$ is equal to

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x-1}$ (C) $\dfrac{1}{x+1}$ (D) $-\dfrac{1}{x+1}$ (E) $-\dfrac{1}{x}$

---

29. For a continuous function $f(x)$ such that $f(0)=6$ and $f(3)=1$, which of the following statements is true?

(A) $f'(x)<0$ for all $x\in(0,3)$

(B) $f'(x)>1$ for all $x\in(0,3)$

(C) there exists $c\in(0,3)$ such that $f'(c)=-\dfrac{5}{3}$

(D) there exists $c\in(1,6)$ such that $f(c)=2$

(E) none of these

30. The graph of $y = 2x^3 - 4x^2 + 2x + 3$ is decreasing and concave upwards when

(A) $\frac{2}{3} < x < 1$

(B) $\frac{1}{3} < x < 1$

(C) $\frac{1}{3} < x < \frac{2}{3}$

(D) $x > 1$

(E) $x < \frac{1}{3}$

---

1. Answer: D

The stationary points of B correspond with the x-intercepts of C.

The stationary points of C correspond with the x-intercepts of A.

2. Answer: D

The limit laws only apply if each individual limit exists.

3. Answer: B

We have

$$f'(x) = \frac{\ln x - 1}{(\ln x)^2}$$

When $f'(x) = 0$ we have

$$x = e$$

So the minimum value is $\frac{e}{\ln e} = e$.

4. Answer: E

We have

$$\frac{dy}{dx} = 2x - 4$$

So the equation of the normal is

$$y + 2 = -\frac{1}{2}(x - 3)$$

Giving

$$y = -\frac{x}{2} - \frac{1}{2}$$

So we need

$$-\frac{x}{2} - \frac{1}{2} = x^2 - 4x + 1$$

Giving

$$2x^2 - 7x + 3 = 0$$

Factorise

$$(2x - 1)(x - 3) = 0$$

It therefore intersects again when $x = \frac{1}{2}$.

5. Answer: B

The average value is

$$\frac{1}{1-(-1)}\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{1}{2}\times\frac{\pi\times 1^2}{2} = \frac{\pi}{4}$$

6. Answer: B

Use the substitution $u = x^2$. So $\dfrac{du}{dx} = 2x$. The integral then becomes

$$\frac{1}{2}\int_0^{\pi} \sin u \, du$$

7. Answer: D

We have

$$A = \pi r^2$$

And

$$C = 2\pi r$$

So

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

And

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

Therefore

$$\frac{dA}{dt} = r\frac{dC}{dt}$$

So

$$r = 10\,\text{cm}$$

8. Answer: C

This is the limit definition of the integral $\int_1^e \ln x \, dx$.

This is equal to

$$[x\ln x - x]_1^e = e - e - (0 - 1) = 1$$

9. Answer: D

We have

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x \to 3} x + 3 = 6$$

10. Answer: B

The product is

$$P = x(30 - x) = -x^2 + 30x = -(x - 15)^2 + 225$$

11. Answer: B

If $y = e^{-x}$ then $\frac{dy}{dx} = -y$. This doesn't match the slope field.

If $y = e^{1/x}$ then $\frac{dy}{dx} = -\frac{y}{x}$. This matches the slope field.

If $y = e^{x^2}$ then $\frac{dy}{dx} = 2xy$. This doesn't match the slope field.

If $y = e^{-1/x}$ then $\frac{dy}{dx} = \frac{y}{x}$. This doesn't match the slope field.

If $y = e^{-x^2}$ then $\frac{dy}{dx} = -2xy$. This doesn't match the slope field.

12. Answer: A

We have

$$\sin y + x\frac{dy}{dx}\cos y = \frac{dy}{dx}\cos x - y\sin x$$

Giving

$$\frac{dy}{dx}(\cos x - x\cos y) = \sin y + y\sin x$$

So

$$\frac{dy}{dx} = \frac{\sin y + y\sin x}{\cos x - x\cos y}$$

We therefore need $\cos x = x\cos y$.

13. Answer: D

We have

$$\ln y + \frac{x}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

So

$$\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = 1 - \ln y$$

Giving

$$\frac{dy}{dx} = \frac{y(1 - \ln y)}{x - y}$$

So we need $x = y$ giving

$$x \ln x = 2x$$

So

$$x = y = e^2$$

14. Answer: D

We have

$$f'(x) = e^{1-\sqrt{x}} - \frac{\sqrt{x}\,e^{1-\sqrt{x}}}{2} = e^{1-\sqrt{x}}\left(1 - \frac{\sqrt{x}}{2}\right)$$

This is equal to zero when $x = 4$.

Since $f(0) = 0$ and $f(x) > 0$ for all $x > 0$ it must be initially increasing.

So it is increasing on the interval $0 < x < 4$.

15. Answer: C

We have

$$\frac{du}{dx} = \sin x \sec^2 x + \sec^2 x = \sec^2 x\,(\sin x + 1)$$

When $x = 0$ we have $u = 1$ and when $x = \pi/3$ we have $u = 2 + \sqrt{3}$.

So the integral becomes

$$\int_1^{2+\sqrt{3}} \frac{1}{\sec x + \tan x}\,du = \int_1^{2+\sqrt{3}} \frac{1}{u}\,du = \ln\left(2 + \sqrt{3}\right)$$

16. Answer: C

We need $v(k) = 0$ so

$$2k^{1/3} = k^{1/2}$$

Giving

$$2 = k^{1/6}$$

So

$$k = 64$$

17. Answer: C

We have

$$\left(y + x\frac{dy}{dx}\right)e^{xy} = 1 + \frac{dy}{dx}$$

So

$$\frac{dy}{dx}(xe^{xy} - 1) = 1 - ye^{xy}$$

Giving

$$\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1}$$

18. Answer: E

In all cases $f(x) \leq h(x)$.

This can be confirmed by attempting to find points of intersection e.g.

$$2x^2 - 3 = 4x - 5$$

Giving

$$2x^2 - 4x + 2 = 0$$

So

$$x^2 - 2x + 1 = 0$$

Giving

$$(x - 1)^2 = 0$$

A repeated solution implies the graphs are tangential, so don't completely cross each other.

19. Answer: C

We have

$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

So the equation becomes

$$x\frac{du}{dx} + u = f(u)$$

Giving

$$x\frac{du}{dx} = f(u) - u$$

So

$$\int \frac{1}{f(u) - u}\,du = \int \frac{1}{x}\,dx$$

20. Answer: A

Let $y = \sec^{-1} x$ so $x = \sec y$. We have

$$\frac{dx}{dy} = \sec^2 y \sin y = \sec^2(\sec^{-1} x)\sqrt{1 - \cos^2(\sec^{-1} x)}$$

This is equal to

$$x^2\sqrt{1 - \frac{1}{x^2}} = \sqrt{x^2(x^2 - 1)}$$

Therefore

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2(x^2 - 1)}}$$

21. Answer: B

The intermediate value theorem guarantees two solutions only when $k = 8$.

21. Answer: B

22. Answer: E

The volume is equal to

$$\pi\int_1^2 (\sqrt{y-1})^2\,dy = \pi\left[\frac{y^2}{2} - y\right]_1^2 = \pi\left(\frac{4}{2} - 2 - \left(\frac{1}{2} - 1\right)\right) = \frac{\pi}{2}$$

23. Answer: B

We have

$$\int_0^1 \frac{1}{x^2+4x+5}\,dx = \int_0^1 \frac{1}{(x+2)^2+1}\,dx = \left[\arctan(x+2)\right]_0^1 = \arctan 3 - \arctan 2$$

24. Answer: C

Use the substitution $u = \frac{x}{2}$. So $\frac{du}{dx} = \frac{1}{2}$. The integral then becomes

$$2\int_1^3 f(u)\,du = 2 \times 8 = 16$$

25. Answer: B

Let

$$y = x(10 + k\ln x)^{1/3}$$

So

$$\frac{dy}{dx} = (10 + k\ln x)^{1/3} + \frac{k}{3(10 + k\ln x)^{2/3}}$$

Substitute into the differential equation

$$(10 + k\ln x)^{1/3} + \frac{k}{3(10 + k\ln x)^{2/3}} = \frac{x^3 + x^3(10 + k\ln x)}{x^3(10 + k\ln x)^{2/3}}$$

Rewrite

$$\frac{3(10 + k\ln x) + k}{3(10 + k\ln x)^{2/3}} = \frac{11 + k\ln x}{(10 + k\ln x)^{2/3}}$$

Therefore

$$k = 3$$

26. Answer: A

We have

$$\int_{-4}^{0} \frac{x^2}{x+5}\,dx = \int_{-4}^{0} x - 5 + \frac{25}{x+5}\,dx = \left[\frac{x^2}{2} - 5x + 25\ln(x+5)\right]_{-4}^{0}$$

This is equal to

$$25\ln 5 - \left(\frac{16}{2} - (-20)\right) = 25\ln 5 - 28$$

27. Answer: C

We have

$$g(x) = F(x) - F(0)$$

where $F'(x) = f(x)$.

So

$$g'(x) = f(x)$$

When $f(x) = 0$ we have $g'(x) = 0$. This corresponds to point C.

28. Answer: E

We have

$$\lim_{h \to 0} \frac{\ln(x-h) - \ln x}{h} = -\lim_{h \to 0} \frac{\ln(x-h) - \ln x}{-h} = -\frac{1}{x}$$

29. Answer: E

Note that it isn't stated whether the function is differentiable.

30. Answer: A

We have

$$\frac{dy}{dx} = 6x^2 - 8x + 2$$

And

$$\frac{d^2y}{dx^2} = 12x - 8$$

We need

$$6x^2 - 8x + 2 < 0$$

So

$$(3x - 1)(x - 1) < 0$$

Giving

$$\frac{1}{3} < x < 1$$

And we need

$$12x - 8 > 0$$

So

$$x > \frac{8}{12} = \frac{2}{3}$$

Therefore

$$\frac{2}{3} < x < 1$$

1	A	B	C	**D**	E
2	A	B	C	**D**	E
3	A	**B**	C	D	E
4	A	B	C	D	**E**
5	A	**B**	C	D	E
6	A	**B**	C	D	E
7	A	B	C	**D**	E
8	A	B	**C**	D	E
9	A	B	C	**D**	E
10	A	**B**	C	D	E
11	A	**B**	C	D	E
12	**A**	B	C	D	E
13	A	B	C	**D**	E
14	A	B	C	**D**	E
15	A	B	**C**	D	E
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17	A	B	**C**	D	E
18	A	B	C	D	**E**
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21	A	**B**	C	D	E
22	A	B	C	D	**E**
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25	A	**B**	C	D	E
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27	A	B	**C**	D	E
28	A	B	C	D	**E**
29	A	B	C	D	**E**
30	**A**	B	C	D	E